QUANTITATIVE RELATIONSHIPS GOVERNING THE ESTABLISHMENT
OF A QUASISTEADY DESTRUCTION REGIME ON UNILATERIAL MATERIAL HEATING

Yu. V. Polezhaev and G. A. Frolov   UDC 536.2.083

The nonsteady period of mass removal (entrainment) from the surface of a material has been investigated both theoretically and experimentally. Analytical relationships are proposed to make possible calculation of the characteristics of the nonsteady destruction regime for body surfaces streamlined by a high-temperature stream of gas.

Experimental comparison of heat-resistant materials in the majority of cases is conducted under conditions of quasisteady destruction, in which a virtually constant value for the entrainment rate $\bar{V}_\infty$ is established. Attainment of the quasisteady value of $\bar{V}_\infty$ occurs after some time τ_V subsequent to the start of heating (Fig. 1).

A formula was proposed in [1] which enables us to calculate τ_V in the presence of constant thermophysical properties with consideration of the thermal efficiency m of the material:

$$\tau_v = \frac{a}{\bar{V}_\infty^2} f(m).$$ (1)

However, application of (1) in actual calculations is exceedingly complicated by the need to determine the coefficient of thermal diffusivity a for the material surface layer and for the dimensionless coefficient of its thermal efficiency m.

The rate of material entrainment under the experimental conditions is frequently found by measuring the linear entrainment to the extent that it is dependent on the heating time. In this case, processing of the results by the method of least squares when $\tau > \tau_V$ enables us to obtain an equation for the linear entrainment in the form

$$S(\tau) = \bar{V}_\infty \tau - d_0,$$ (2)

where d_0 is a parameter which denotes the displacement of the straight line of linear entrainment relative to the coordinate origin.

For purposes of experimentally determining the parameter d_0, we investigated both doped and pure quartz glazed ceramics, quartz glass, plastic with a glass filler, and asbestos textolite, all subjected to heating by radiation, convection, and a combination of radiation and convection. The experiment lasted anywhere from 1 to 60 sec. The density of the heat flux varied from 6000 to 50,000 kW/m^2, while the deceleration enthalpy ranged from 4000 to 40,000 kJ/kg. As we can see from Table 1, the parameter d_0 depends only slightly on the heating conditions. To estimate the effect exerted by the coefficient of thermal conductivity for the material surface layer on d_0 with identical heat flow (7650 kW/m^2), we tested specimens of quartz glass, of pure and doped quartz glazed ceramics (Fig. 1). As the transparency of the material was increased (the doped and pure ceramic, the quartz glass), which is equivalent to increasing the coefficient of thermal conductivity by means of the radiant component of heat transfer, the value of d_0 increases. Analogous results are achieved through the presence in the material of a binder (glass plastics, asbestos textolite; Table 1). On decomposition of the binder, a porous coke structure is formed, and here the radiant component of heat transfer also increases.

In order theoretically to determine the parameter d_0, we used the relationship between the linear entrainment and the heating time $S(\tau)$, obtained by means of the melting model described in [1]. During the course of our calculations, the values of density, viscosity,
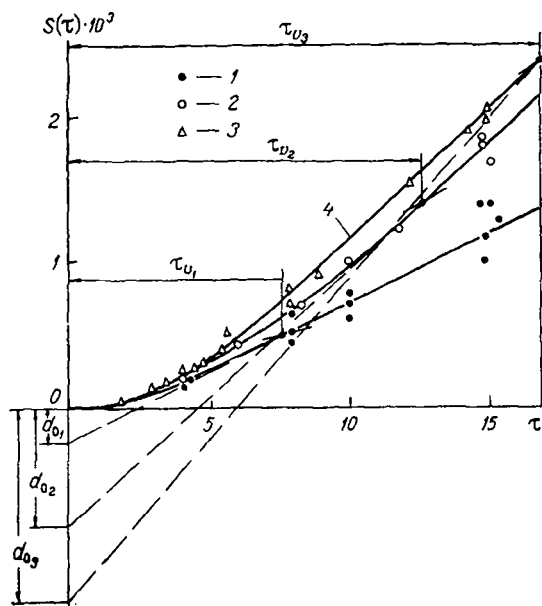
Fig. 1. The time required to establish the quasisteady rate of entrainment as a function of the parameter d_0 with an identical heat flux of ~7650 kW/m²: 1-3) experiment; 4) calculation on the basis of (4); 1) doped quartz ceramic, $\bar{V}_\infty = 0.1 \cdot 10^{-3}$ m/sec; 2) pure quartz ceramic, $\bar{V}_\infty = 0.17 \cdot 10^{-3}$ m/sec; 3) quartz glass, $\bar{V}_\infty = 0.22 \cdot 10^{-3}$ m/sec, $S(\tau)$, m; τ, sec.
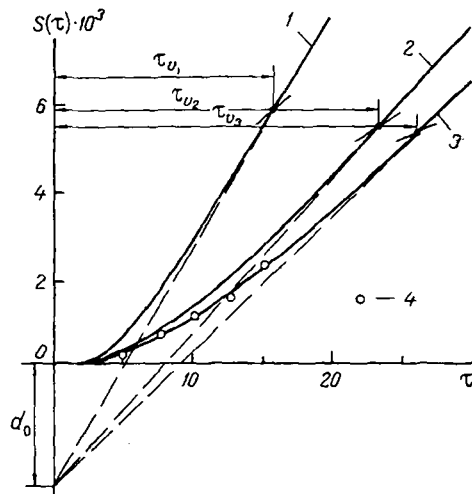

Fig. 2. The linear entrainment of quartz glass ($\lambda = 10.4$ W/m·K) as a function of the heating time for $P_e = 0.13 \cdot 10^5$ Pa, $I_e = 15,000$ kJ/kg: 1-3) calculations based on the model described in [1], and formula (4); 4) experiment [6]; 1) $\mu = \exp[(68,800/T_w) - 24.59]$; 2) $\exp[(50,500/T_w) - 15.0]$; 3) $9.8 \exp[(\exp\{1180/T_w + 10.05\}/T_w) - 9.5]$, (kg·sec)/m².

heat capacity, the vaporization fraction, the pressure gradient, aerodynamic friction, emissivity, heat of vaporization, and thermal conductivity all varied over a wide range. For example, the heat capacity varied by a factor of 2, the vaporization fraction varied by a factor of 10, the density varied by a factor of 1.5, etc. Despite the fact that the entrainment rate here changed approximately by a factor of 4, while the surface temperature ranged from 2400 to 2840 K, the parameter d_0 in virtually all of the cases was equal to $(1.1 \pm 0.1) \cdot 10^{-3}$ m. These calculations confirmed that d_0 depends primarily on the coefficient of material thermal conductivity, whose change from 1.7 to 4.2 W/m·K resulted in an increase of d_0 from $0.9 \cdot 10^{-3}$ to $1.8 \cdot 10^{-3}$ m (Table 2).

TABLE 1. Results of Experimental Determination of the Parameter d_0 under Various Heating Conditions

Material	Heat flux, kW/m² q_{con}	Heat flux, kW/m² q_{rad}	T_w, K	$\overline{V}_\infty \cdot 10^2$, m/s	$d_0 \cdot 10^3$, m	$d_0^{av} \cdot 10^3$, m
Doped ceramic (0,5% Cr_2O_3)	7650	—	2620	0,1	0,26	
	11 500	—	2800	0,18	0,29	
	14 000	—	2820	0,21	0,26	0,26±10%
	10 500	+10500	2840	0,5	0,24	
	50 000	—	~3000	0,91	0,23	
Glass plastic with epoxy binder	6000	—	2610	0,21	0,81	
	8500	—	2650	0,35	0,92	
	14 000	—	2690	0,52	0,84	
	—	2100	2270	0,075	0,9	0,81±14%
	—	6300	2730	0,24	0,77	
	—	10 500	2860	0,33	0,7	
	—	14 700	2860	0,47	0,76	
Asbestos-treated textolite	—	2100	2160	0,03	0,4	
	2100	—	2160	0,066	0,36	
	5000	—	2760	0,20	0,45	
	9400	—	2980	0,25	0,4	0,4±15%
	11 600	—	3030	0,29	0,42	
	50 000	—	—	1,75	0,34	

TABLE 2. Results of Numerical Calculations of the Parameter d_0 and Determination of the Establishment Time τ_v and of Linear Entrainment $S(\tau_v)$ from Condition (7), as Well as on the Basis of Formulas (8) and (9)

Parameter being studied — designation	value	$\overline{V}_\infty \cdot 10^3$, m/sec	$d_0 \cdot 10^3$, m	τ_v, sec (7)	τ_v, sec (8)	$S(\tau_v) \cdot 10^3$, m (7)	$S(\tau_v) \cdot 10^3$, m (9)
c_p, kJ/(kg·K)	1,26	0,41	1,1	7,6	7,6	2,0	2,0
	1,51	0,36	1,0	8,0	7,9	1,8	1,8
	1,01	0,495	1,2	6,6	6,9	2,15	2,2
	2,51	0,23	0,8	9,0	9,8	1,3	1,5
$c_p = a + b(T - T_0)$	$\dfrac{1,26^*}{1,05 \cdot 10^{-3}}$	0,24	0,9	10,0	10,6	1,6	1,64
ρ, kg/m³	2400	0,36	1,15	9,5	9,0	2,1	2,1
	1600	0,5	1,15	6,4	6,5	2,1	2,1
d^2P_e/dx^2, N/m⁴	1·10⁹	0,38	1,1	7,9	8,2	2,0	2,0
	3·10⁹	0,43	1,1	6,7	7,2	1,9	2,0
$d\tau_w/dx$, N/m³	1,4·10⁵	0,39	1,1	7,4	8,0	1,9	2,0
	4,2·10⁵	0,43	1,1	7,1	7,2	2,05	2,0
ΔQ_w, kJ/kg	9220	0,43	1,15	7,2	7,6	2,1	2,1
	10 000	0,425	1,15	7,4	7,7	2,15	2,1
$\varepsilon\sigma T_w^4$, kW/m²	1420	0,34	1,15	9,4	9,6	2,05	2,1
$\mu = \exp\left(\dfrac{a}{T_w} - b\right)$, (kg·sec)/m²		0,89	1,0	2,7	3,2	1,6	1,8
	$\dfrac{55000^*}{24,59}$	0,375	1,15	8,4	8,7	2,0	2,1
	$\dfrac{68\,800^*}{24,09}$	0,58	1,1	5,3	5,4	2,0	2,0
$P_v = \exp\left(-\dfrac{a}{T_w} + b\right) \cdot 10^5$, Pa	$\dfrac{57\,800^*}{14,78}$						
P_e, Pa	2,0	0,465	1,1	6,3	6,7	1,95	2,0
λ, W/(m·K)	1,68	0,37	0,9	7,0	6,9	1,7	1,65
	2,5	0,45	1,3	8,2	8,2	2,35	2,4
	4,2	0,54	1,8	9,4	9,4	3,3	3,3

*The numerator of the fraction gives the value of a, and the denominator gives the value of b.

If the parameter d_0 is comparatively easily determined by means of the theoretical or experimental relationships (2), significant difficulties arise in the determination of the time τ_V, and these are associated with the exceedingly smooth transition of the nonsteady mass entrainment period to one that is stationary. It was proposed in [1] to take as the time required for the establishment of the quasisteady destruction regime for the surface that instant of time in which the mass entrainment rate differs from its steady-state value for $\bar{V}_\infty$ by 5 or 10%.

An interesting method was proposed in [2] to determine the instant r_V of establishment under conditions of erosion surface destruction. It was demonstrated in that reference that the actual rate of mass entrainment at the instant of establishment actually exceeds the average rate in the nonsteady segment by a factor of two. Removal of material in this case is well described by a relationship of the form

$$m_{er(i)} = K m_{p(i)}^2. \tag{3}$$

The results obtained in [2] make it possible for us to establish the instant at which the nonsteady process is concluded on the basis of the intersection of two straight lines: the straight line emanating from the coordinate origin, the tangent of whose slope is equal to $1/2\bar{V}_\infty$, and the straight line described by Eq. (2).

It had been demonstrated earlier [3] that the experimental relationships between the thickness of the removed layer and the time are in satisfactory agreement with calculation on the basis of the expression

$$S(\tau) = \left\{ \frac{\sqrt{\tau} - \sqrt{\tau_{ent}}}{\sqrt{\tau_v} - \sqrt{\tau_{ent}}} \right\}^2 S(\tau_v), \tag{4}$$

where

$$\tau_v = \frac{K_{Td}^2 a}{4\bar{V}_\infty^2}; \tag{5}$$

$$S(\tau_v) = \frac{K_{Td}^2}{4(K_{Td}^2 + 1)} \frac{a}{\bar{V}_\infty}. \tag{6}$$

Comparison of Eqs. (5) and (6) tells us that the average rate of entrainment at the instant τ_V of establishment falls under quantitative relationships similar to those determined in [2] for erosion destruction:

$$\frac{S(\tau_v)}{\tau_v} = V_{av} = \frac{1}{K_{Td}^2 + 1} \bar{V}_\infty \approx 0.65 \bar{V}_\infty. \tag{7}$$

Relationship (7) allows us to determine, rather rigorously, the establishment time τ_V in the case of heat destruction of the material.

When d_0 increases, this results in an increase in the establishment time τ_V, even as the entrainment rate increases. For example, Fig. 1 shows that because of the semitransparency of the quartz glass its entrainment rate and the parameter d_0 are larger than the doped quartz glazed ceramics, by factors of 2 and ~5, respectively. In this case, the time τ_V increases by a factor of ~2. Indeed, since Eq. (2) is valid from the instant of time τ_V, after substitution of (5) and (6), we obtain the expressions

$$\tau_v = \frac{K_{Td}^2 + 1}{K_{Td}^2} \frac{d_0}{\bar{V}_\infty} = 2.83 \frac{d_0}{\bar{V}_\infty}, \tag{8}$$

$$S(\tau_v) = d_0 / K_{Td}^2. \tag{9}$$

Table 2 covers the influence exerted by various parameters on the magnitude of d_0, τ_V, and $S(\tau_V)$. The first version of the calculation shown in this table was carried out for the following parameter values: $P_e = 1 \cdot 10^5$ Pa, $T_e = 4000$ K, $(\alpha/c_p)_0 = 1$ kJ/(m$^2 \cdot$sec), $c_p = 1.26$ kJ/(kg$\cdot$K), $\rho = 2000$ kg/m^3, $\Delta Q_w = 12{,}570$ kJ/kg, $d^2 P_e/dx^2 = 2 \cdot 10^9$ N/m^4, $d\tau_w/dx = 2.8 \cdot 10^5$ N/m^3, $\varepsilon \sigma T_w^4 = 312$ kW/m^2, $\mu = \exp(68{,}000/T_w - 24.59)$ kg$\cdot$sec/m^2, $P_v = \exp[(-57{,}800/T_w) + 18.48] \cdot 10^5$ Pa, $\lambda = 2.1$ W/(m$\cdot$K). In the subsequent versions of the calculation only one of these parameters was changed.
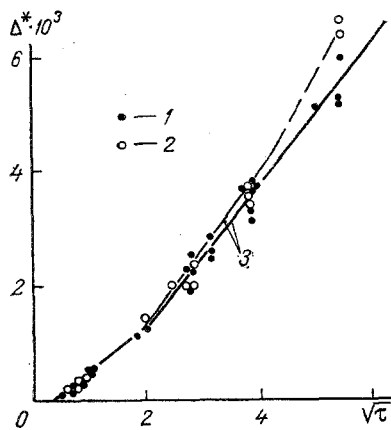
Fig. 3   Fig. 4

Fig. 3. Combined thickness of the heated and removed quartz-glass ceramic layers as a function of the heating time for a heat flux of 7650 kW/m² when T* = 1800 K: 1) doped ceramic; 2) pure ceramic; 3) processing in accordance with the method of least squares (the dashed line represents the quasisteady heating regime). Δ^*, m; $\sqrt{\tau}$, sec.
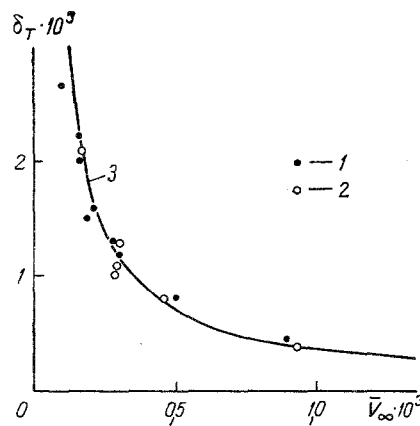
Fig. 4. Depth of heated layer (T* = 1800 K) as a function of the rate of entrainment for doped and pure quartz glass ceramics: 1, 2) experiment; 3) calculation based on (10); 1) doped; 2) pure. δ_T, m; $\bar{V}_\infty$, m/sec.

TABLE 3. The Values of the Establishment Times τ_V and of the Linear Entrainment Time $S(\tau_V)$, Determined on the Basis of the Experimental Functions $S(\tau)$

Materials	q_{con} kW/m²	$\bar{V}_\infty \cdot 10^3$, m/sec	$d_0 \cdot 10^3$, m	τ_v, sec		$S(\tau_v) \cdot 10^3$, m	
				(7)	(8)	(7)	(9)
Doped quartz ceramic	7650	0,1	0,26	7,6	7,4	0,5	0,48
Pure quartz ceramic	7650	0,17	0,85	12,6	14,1	1,4	1,6
Quartz glass	7650	0,22	1,35	17,0	17,4	2,4	2,5
	11 500	0,29	1,1	11,5	10,7	2,15	2,0
Glass plastic with epoxy binder	6000	0,21	0,81	10,7	10,9	1,5	1,5
	14 000	0,52	0,84	4,8	4,6	1,6	1,5

We can see from Table 2 that the values of τ_V and $S(\tau_V)$, determined from condition (7) on the basis of the theoretical relationships $S(\tau)$ (Fig. 2), are in good agreement with the calculations based on formulas (8) and (9). This conclusion also confirms the experimental results presented in Table 3. It is interesting to note that the results of the numerical calculations of linear entrainment, based on the model from [1], completely coincide with the calculations on the basis of (4), if for the determination of τ_V and $S(\tau_V)$ we use Eqs. (8) and (9) (Fig. 2).

Consequently, we can recommend relationships (4), (8), and (9) for the calculation of the parameters of the nonsteady period of thermal destruction in the case of a constant value for the incoming flow of heat.

The test results obtained with doped and pure quartz glazed ceramics in a heat flux of 7650 kW/m² in an electric-arc gas jet heater [4] serve as an excellent illustration of the relationship between the entrainment rate, its time of establishment, the coefficient of thermal diffusivity, and the parameter d_0. In Fig. 3 we see that up to ~15 sec into the heating, the path covered by the isotherm T* = 1800 K from the original surface, both for pure and doped ceramics, is virtually identical. However, because of radiative heat transfer from the surface layer, the rate of entrainment in the case of the pure ceramic is greater

by 70% than in the case of the doped ceramic (see Fig. 1). Since the depth of the heated layer in the pure glazed ceramic in this case diminishes in direct proportion to the increase in its entrainment rate, while the overall thickness Δ^* increases virtually identically in both cases, then the coefficient a, calculated on the basis of the measured heating depth, coincides for both the doped and the pure glazed ceramics (Fig. 4):

$$\delta_T = \frac{a}{\bar{V}_\infty} \ln \frac{T_w - T_0}{T^* - T_0}. \tag{10}$$

Hence we can see that the establishment time τ_V to a considerably greater extent depends on d_0 than on the value of the effective coefficient of thermal diffusivity which determines the depth of the heated layer. Apparently only consideration of the thermophysical properties of the materials directly in the surface layer ($T^* > 1800$ K), including the radiant component of heat transfer, allows us to calculate the establishment time for the entrainment rate in the case of a pure ceramic.

Thus, the increase in the parameter d_0, and consequently the time τ_V, in comparison with its value for a nontransparent uniform material is fundamentally associated with the removal of heat from the surface layer that is due to the radiant heat-transfer component, a consequence either of the transparency or porosity of the material. Since these very factors lead to an increase in the mass entrainment rate [5], the measurement of d_0, τ_V, and $S(\tau_V)$ provides additional information regarding the relationship between the thermophysical properties and temperature directly within the surface layer of the material subjected to destruction.

NOTATION

$\bar{V}_\infty$, the quasisteady value of the velocity of surface displacements; τ, the heating time; τ_V, the establishment time for the quasisteady value of the entrainment rate; τ_{ent}, the time of entrainment onset; a, the coefficient of thermal diffusivity; m, the parameter of the thermal efficiency of the material; $S(\tau)$, the linear entrainment; $S(\tau_V)$, the value of the linear entrainment at the instant at which the quasisteady velocity of surface displacement is established; d_0, the parameter which identifies the displacement of the linear entrainment straight line relative to the coordinate origin; m_{er}, the erosion mass entrainment; m_p, the mass of precipitated particles; K, a constant; K_{T_d}, the destruction constant; V_{av}, is the average entrainment rate in the nonsteady segment; P_e and T_e, the pressure and temperature of deceleration; $(\alpha/c_p)_0$, the heat-exchange coefficient; ρ, density; ΔQ_w, denotes the thermal effect of the phase and physicochemical conversions at the surface; dP_e/dx, the pressure gradient at the leading critical points; τ_w, frictional stress; ε, emissivity; σ, the Stefan–Boltzmann constant; μ, viscosity; P_v, the vapor pressure; λ, the coefficient of thermal conductivity; T_w, the temperature of the heated surface; T^*, the isotherm temperature; q_{con} and q_{rad}, the convection and radiation calorimetric heat flows.

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